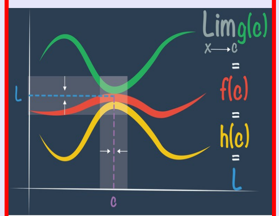
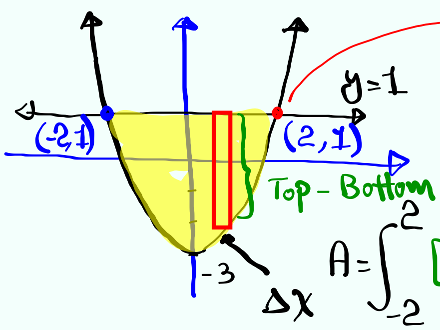
Calculus I

Lecture 51



Feb 19-8:47 AM

Find the area bounded by $y=1$ and $y=x^2-3$.



$$\begin{cases} y=1 \\ y=x^2-3 \end{cases} \rightarrow x^2-3=1$$

$$x^2=4$$

$$x=\pm 2$$

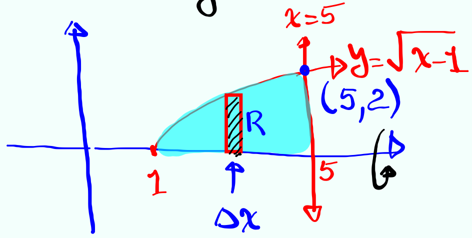
$$A=\int_{-2}^{2}\left[1-(x^2-3)\right]dx$$

$$=2\int_0^2 (4-x^2)\,dx = 2\left[4x-\frac{x^3}{3}\right]\Big|_0^2 = \boxed{}$$

Dec 4-7:26 AM

$y=\sqrt{x}$

Rotate the region bounded by $y=\sqrt{x-1}$, $y=0$, and $x=5$ by the x-axis. Find its volume.



1) Ref. Rect. ⊥ A.O.R.

2) Region is totally attached to A.O.R.

Disk Method

$R = y_{curve} = \sqrt{x-1}$

$$V=\int_1^5 \pi\left[R(x)\right]^2 dx = \pi\int_1^5 \left(\sqrt{x-1}\right)^2 dx$$

$$=\pi\int_1^5 (x-1)\,dx$$

$$=\pi\left[\frac{x^2}{2}-x\right]\Big|_1^5 = \boxed{}$$

Dec 4-7:33 AM

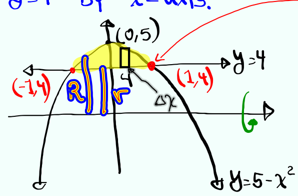
Rotate the region bounded by $y=5-x^2$ and $y=4$ by x-axis.

$\begin{cases} y=5-x^2 \\ y=4 \end{cases} \Rightarrow 5-x^2=4$

$5-4=x^2$

$x^2=1$

$x=\pm 1$



1) Ref. Rect. ⊥ A.O.R.

2) Region is not totally attached to A.O.R.

Washer Method

$R = y_{curve} = 5-x^2$

$r = y_{line} = 4$

$$V=\int_{-1}^{1} \pi\left[R^2-r^2\right]dx$$

$$=\int_{-1}^{1} \pi\left[(5-x^2)^2-4^2\right]dx$$

$$=2\pi\int_0^1 \left[25-10x^2+x^4-16\right]dx$$

$$=2\pi\int_0^1 \left[9-10x^2+x^4\right]dx = 2\pi\left[9x-\frac{10x^3}{3}+\frac{x^5}{5}\right]\Big|_0^1$$

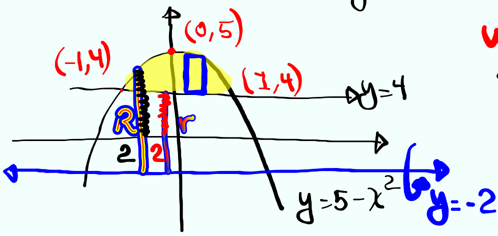
$$=2\pi\left[9-\frac{10}{3}+\frac{1}{5}-0\right]$$

$$=\boxed{\frac{176\pi}{15}}$$

Dec 4-7:40 AM

Set-up only:

Take the bounded region from last example and rotate about $y=-2$. Find the volume.



Washer Method

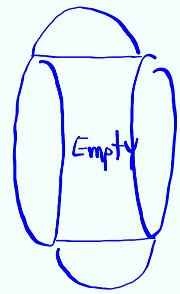
$R = y_{curve} + 2$

$r = y_{line} + 2$

$R = 5 - x^2 + 2 = 7 - x^2$

$r = 4 + 2 = 6$

$$V = \int_{-1}^{1} \pi\left[(7-x^2)^2 - 6^2\right] dx$$



Dec 4-7:53 AM

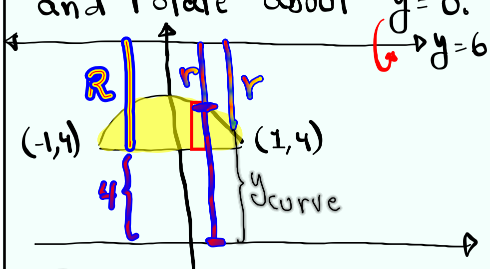
Set-up only:

Take the bounded region from last example and rotate about $y=6$.

Washer Method

$R = 2$

$r = 1 + x^2$



$R + 4 = 6 \qquad R = 6 - 4 = 2$

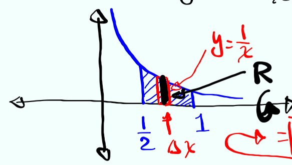
$r + y_{curve} = 6 \qquad r = 6 - y_{curve} = 6 - (5 - x^2) = 1 + x^2$

$$V = \int_{-1}^{1} \pi\left[2^2 - (1+x^2)^2\right] dx$$

Dec 4-8:02 AM

Consider the region bounded by $y=\frac{1}{x}$, $y=0$, $x=\frac{1}{2}$ and $x=1$.

1) Draw it



$= \boxed{\ln 2}$

2) Find its area

$$A=\int_{1/2}^{1}\frac{1}{x}\,dx=\ln x\Big|_{1/2}^{1}=\cancel{\ln 1}^{\,0}-\ln \tfrac{1}{2}=-(\cancel{\ln 1}^{\,0}-\ln 2)$$

3) Find the volume if rotated about x-axis.

Disk Method

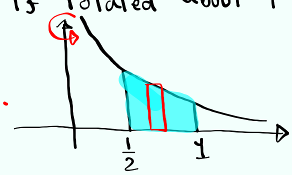
$$V=\int_{1/2}^{1}\pi\left[\frac{1}{x}\right]^2dx=\pi\int_{1/2}^{1}\frac{1}{x^2}dx$$

$$=\pi\int_{1/2}^{1}x^{-2}dx=\pi\cdot\frac{x^{-1}}{-1}=-\pi\cdot\frac{1}{x}\Big|_{1/2}^{1}=\boxed{}$$

4) Find the volume if rotated about y-axis.

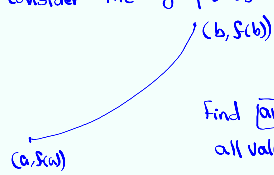
Since Ref. Rect. is Parallel to the A.O.R.

⇒ Shell Method



Dec 4-8:10 AM

Consider the graph of $y=f(x)$ over $[a,b]$ → must be cont. on $[a,b]$



Find $\underbrace{\boxed{\text{average value}}}$ (y_{ave}) of all values of $f(x)$ over $[a,b]$

$$y_{ave}=\frac{y_1+y_2+y_3+\cdots+y_n}{n}$$

$$y_{ave}=\frac{f(x_1)+f(x_2)+f(x_3)+\cdots+f(x_n)}{n}$$

we want $n\to\infty$

$$=\frac{\sum_{i=1}^{n}f(x_i)}{n}$$

Recall

$\Delta x=\frac{b-a}{n}$

Solve for n

$n=\frac{b-a}{\Delta x}$

$$y_{ave}=\lim_{n\to\infty}\frac{\sum_{i=1}^{n}f(x_i)}{n}$$

$$y_{ave}=\lim_{n\to\infty}\frac{\sum_{i=1}^{n}f(x_i)}{\frac{b-a}{\Delta x}}=\lim_{n\to\infty}\frac{1}{b-a}\sum_{i=1}^{n}f(x_i)\Delta x$$

$$y_{ave}=\frac{1}{b-a}\boxed{\lim_{n\to\infty}\sum_{i=1}^{n}f(x_i)\Delta x}$$

$$=\frac{1}{b-a}\int_a^b f(x)\,dx$$

Dec 4-8:23 AM

Find f_{ave} for $f(x) = x^2$ on $[0, 1]$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{1-0}\int_0^1 x^2\,dx$$

$$= 1 \cdot \frac{x^3}{3}\Big|_0^1 = \boxed{\frac{1}{3}}$$

Dec 4-8:33 AM